**The Theory of Numerical :**

1. **Solution of Non-linear Equations:**
2. **Bisection:**

#### The Bisection Method is a successive approximation method that narrows down an interval that contains a root of the function f(x)

#### The Bisection Method is given an initial interval [a..b] that contains a root

#### (We can use the property sign of f(a) ≠ sign of f(b) to find such an initial interval)

#### The Bisection Method will cut the interval into 2 halves and check which half interval contains a root of the function

#### The Bisection Method will keep cut the interval in halves until the resulting interval is extremely small

#### The root is then approximately equal to any value in the final (very small) interval.

1. **Regular Falsi:**

The [regula-falsi method](https://math.iitm.ac.in/public_html/sryedida/caimna/transcendental/bracketing methods/regula-falsi/regula-falsi.html" \t "https://www.goseeko.com/blog/what-is-regula-falsi-method/_blank) is the oldest method of finding the approximate numerical value of a real root of an equation f(x) = 0.

This method is also known as [method of false position](https://math.iitm.ac.in/public_html/sryedida/caimna/transcendental/bracketing methods/regula-falsi/regula-falsi.html" \t "https://www.goseeko.com/blog/what-is-regula-falsi-method/_blank). The method used to estimate the roots of a polynomial f(x).

In this method we suppose that x1 and x2 are two points where f(x1) and f(x2) are of opposite sign .Let f(x1) < 0 and f(x2) > 0.

Hence the root of the equation f(x) = 0 lies between x1 and x1 and so, f(x1)f(x2) < 0.

Formula of regula-falsi method

The Regula Falsi formula :

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1. **Secant:**

 The [secant](https://www.sciencedirect.com/topics/mathematics/secant" \o "Learn more about secant from ScienceDirect's AI-generated Topic Pages) method is similar to the Newton-Raphson method in that a straight line is used to determine the next approximation to the root. In contrast to the Newton-Raphson method, the secant method uses two initial guesses for the root, x0 and x1(≠x0), and a straight line is fitted between the evaluations of f(x) at these positions. This line is called the secant line and an approximation of the root, x2, is given by the intercept of the secant line with the x-axis.

1. **Newton Raphson:**

Newton Raphson Method is yet another numerical method to approximate the root of a polynomial. Newton Raphson Method is an open method of root finding which means that it needs a single initial guess to reach the solution instead of narrowing down two initial guesses. Newton Raphson Method uses to the slope of the function at some point to get closer to the root. Using equation of line **y =**[m](javascript:window.alert("Help: m is the slope at a given point (first derivative)")" \o "Help: m is the slope at a given point (first derivative))**x0 + c** we can calculate the point where it meets x axis, in a hope that the original function will meet x-axis somewhere near.

1. **Fixed Point:**

Fixed point iteration method is open and simple method for finding real root of non-linear equation by successive approximation. It requires only one initial guess to start. Since it is open method its convergence is not guaranteed. This method is also known as **Iterative Method.** To find the root of nonlinear equation f(x)=0 by fixed point iteration method, we write given equation f(x)=0 in the form of x = g(x).

1. **Interpolation and Approximation:**
2. **Lagrange:**

The Lagrange interpolation formula is a way to find a polynomial, called Lagrange polynomial, that takes on certain values at arbitrary points. Lagrange’s interpolation is an Nth degree polynomial approximation to f(x).

Given n distinct real values x1,x2,…,xnx1,x2,…,xn​ and n real values y1,y2,…,yny1,y2,…,yn (not necessarily distinct), there is a unique polynomial P with real coefficients satisfying P(xi)=yiP(xi)=yi for  i ∈ {1, 2, …, n}, such that deg(P) < n.

1. **Least Square Method:**
2. **Linear:**

Used directly, with an [appropriate data set](https://www.itl.nist.gov/div898/handbook/pmd/section3/pmd32.htm), linear least squares regression can be used to fit the data with any function of the form

f(x⃗;β⃗)=β0+β1x1+β2x2+…

in which

each explanatory variable in the function is multiplied by an unknown parameter,

there is at most one unknown parameter with no corresponding explanatory variable, and

all of the individual terms are summed to produce the final function value.

1. **Exponential:**
2. **Polynomial:**
3. **Differentiation and Integration:**
4. **Trapezoidal Rule: (with function and data)**

Trapezoidal Rule is a rule that evaluates the area under the curves by dividing the total area into smaller trapezoids rather than using rectangles. This integration works by approximating the region under the graph of a function as a trapezoid, and it calculates the area. This rule takes the average of the left and the right sum.

The Trapezoidal Rule does not give accurate value as Simpson’s Rule when the underlying function is smooth. It is because Simpson’s Rule uses the quadratic approximation instead of linear approximation. Both Simpson’s Rule and Trapezoidal Rule give the approximation value, but [Simpson’s Rule](https://byjus.com/maths/simpsons-rule/) results in even more accurate approximation value of the integrals.

Trapezoidal Rule Formula

Let f(x) be a continuous function on the interval [a, b]. Now divide the intervals [a, b] into n equal subintervals with each of width,

Δx = (b-a)/n, Such that a = x0 < x1< x2< x3<…..<xn = b

1. **Simpson’s 1/3 Rule: (with function)**

Simpson’s 1/3rd rule is an extension of the trapezoidal rule in which the integrand is approximated by a second-order polynomial. Simpson rule can be derived from the various way using Newton’s divided difference polynomial,  Lagrange polynomial and the method of coefficients. Simpson’s 1/3 rule is defined by:

|  |
| --- |
| ∫ab f(x) dx = h/3 [(y0 + yn) + 4(y1 + y3 + y5 + …. + yn-1) + 2(y2 + y4 + y6 + ….. + yn-2)] |

This rule is known as Simpson’s****One-third rule****.

1. **Simpson’s 3/8 Rule:(with function)**

Another method of numerical integration is called “Simpson’s 3/8 rule”. It is completely based on the cubic interpolation rather than the quadratic interpolation. Simpson’s 3/8 or three-eight rule is given by:

|  |
| --- |
| ∫ab f(x) dx = 3h/8 [(y0 + yn) + 3(y1 + y2 + y4 + y5 + …. + yn-1) + 2(y3 + y6 + y9 + ….. + yn-3)] |

This rule is more accurate than the standard method, as it uses one more functional value. For 3/8 rule, the composite Simpson’s 3/8 rule also exists which is similar to the generalized form. The 3/8 rule is known as Simpson’s second rule of integration.

1. **Solution of System Linear Algebraic Equations:**
2. **Gauss Jordan:**
3. **Gauss Elimination:**

**Gauss-Jordan Elimination** is an algorithm that can be used to solve systems of linear equations and to find the inverse of any invertible matrix. It relies upon three **elementary row operations** one can use on a matrix:

1. Swap the positions of two of the rows
2. Multiply one of the rows by a nonzero scalar.
3. Add or subtract the scalar multiple of one row to another row.

The purpose of Gauss-Jordan Elimination is to use the three elementary row operations to convert a matrix into reduced-row echelon form. A matrix is in **reduced-row echelon form**, also known as **row canonical form**, if the following conditions are satisfied:

1. All rows with only zero entries are at the bottom of the matrix
2. The first nonzero entry in a row, called the **leading entry** or the **pivot**, of each nonzero row is to the right of the leading entry of the row above it.
3. The leading entry, also known as the pivot, in any nonzero row is 1.
4. All other entries in the column containing a leading 1 are zeroes.
5. **Matrix Inverse:**
6. **Gauss Seidal:**
7. **Power Method:**
8. **Ordinary Differential Equations:**

In mathematics, the term “**Ordinary Differential Equations**” also known as **ODE** is an equation that contains only one independent variable and one or more of its derivatives with respect to the variable. In other words, the ODE is represented as the relation having one independent variable x, the real dependent variable y, with some of its derivatives.

y’,y”, ….yn ,…with respect to x.

### Order

The order of ordinary differential equations is defined to be the order of the highest derivative that occurs in the equation. The general form of n-th order ODE is given as;

F(x, y,y’,….,yn ) = 0

1. **Euler’s:(1st order IVP-initial value problem)**
2. **RK2:(1st order)**
3. **RK4:(1st order)**
4. **RK4:(1st order Simultaneous)**
5. **ShootingRK4:(2nd order)**